A nonperturbative determination of C_A

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We describe a non-perturbative determination of c_A using correlators containing the axial-vector and pseudoscalar currents at zero and non-zero momentum. We apply the method of Bhattacharya et al to extract c_A from the requirement that the ratio of appropriate correlators for the PCAC relation becomes independent of time in the excited state region. We find that the result depends strongly on the order of the derivatives used in the PCAC relation. We also find that, using the lowest order derivatives, we cannot get a consistent value of c_A between zero and non-zero momentum cases. The c_A values that we obtain as we improve the derivatives are consistent and decrease in magnitude heading towards the perturbative result.

1. Introduction

The use of Symanzik-improved lattice actions and matrix elements is widespread. However, with each improvement term added the corresponding coefficient must be determined to enable discretisation effects to be reduced to the desired level. Considering the light hadron spectrum and matrix elements, the relevant $O(a)$ improvement coefficients are, for the most part, only known to 1-loop in perturbation theory. A nonperturbative determination of these coefficients is desirable and may be simpler than performing higher loop perturbative calculations.

The ALPHA collaboration, using Schrödinger functional techniques, have calculated several $O(a)$ improvement coefficients nonperturbatively. In most cases agreement is found with 1-loop (tadpole-improved) perturbation theory, or the discrepancy is consistent with estimates of the size of omitted higher orders in α (albeit in some cases assuming a slow convergence of the perturbative series). However, for the $O(a)$ improvement of the axial-vector current

$$A_\mu \rightarrow A_\mu^I = A_\mu + c_A \partial_\mu P + O(a^2) \qquad (1)$$

where $A_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi$ and $P = \bar{\psi}\gamma_5\psi$, c_A is found to be many times larger than the 1-loop perturbative value even at reasonably high β, as shown in figure 1, taken from reference [1].

c_A appears in the expression for the pion decay constant, f_π,

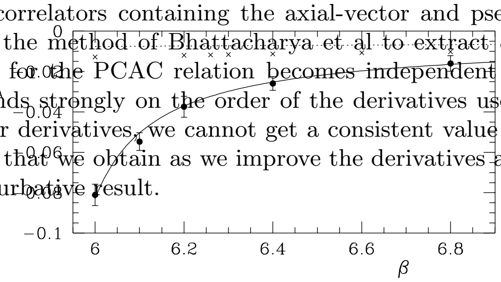

Figure 1. C_A extracted by the ALPHA collaboration. The circles indicate the results of reference [1], the dotted line the results of 1-loop perturbation theory and the crosses the results from 1-loop tadpole-improved perturbation theory.

$$f_\pi = (1 + b_A) Z_A (f_\pi^{(0)} + c_A a f_\pi^{(1)} + O(a^2) \qquad (2)$$

and the bare quark mass (from the PCAC relation)

$$2m = \frac{\partial_\mu < A_4 P > + c_A a \partial_\mu^2 < PP >}{2 < PP >} + O(a^2) \qquad (3)$$

Choosing the perturbative or nonperturbative value for c_A leads to significantly different values for these quantities at finite β. Bhattacharya et. al. proposed an alternative method for determining c_A, and other $O(a)$ coefficients in reference [2]. Here, we apply their method, but with higher statistics than reported in reference [2]. In addition, we extract c_A at finite, as well as zero, momentum.

2. C_A from the PCAC relation

The PCAC relation in euclidean space

$$< \partial_\mu A_\mu(x) P >= 2m < P(x) P > \qquad (4)$$

should hold for all x on the lattice up to discretisation terms. For simplicity we assign

$$r(t) = \frac{< \partial_\mu A_\mu(t) P >}{< P(t) P >} \quad (5)$$

$$s(t) = \frac{< \partial_\mu^2 P(t) P >}{< P(t) P >} \quad (6)$$

Thus,

$$2m = r(t) + \mathrm{O(a)} \quad (7)$$

$$2m = r(t) + c_A s(t) + O(a^2). \quad (8)$$

Equation 8 only holds if the $O(a)$-improvement term (the clover term) is included in the light quark action and the coefficient, c_{SW}, is determined nonperturbatively. The method of Bhattacharya et. al. is to determine c_A by minimizing the dependence of $r(t) + c_A s(t)$ on t and, hence, reduce the discretisation errors in this quantity. Using this method, we investigate the effect of improving ∂_μ on the determination of c_A; discretisation errors in the lattice derivatives may be the dominant source of lattice spacing errors in the bare quark mass, and hence, the value obtained for c_A may be artificially high.

Normally, the symmetric lattice derivatives

$$\partial_\mu \to \Delta_\mu^{(+-)} = \frac{1}{2}(\delta_{\vec{x},\vec{x}+\hat{\mu}} - \delta_{\vec{x},\vec{x}-\hat{\mu}}) \quad (9)$$

$$\partial_\mu^2 \to \Delta_\mu^{(2)} = \delta_{\vec{x},\vec{x}+\hat{\mu}} - 2\delta_{\vec{x},\vec{x}} + \delta_{\vec{x},\vec{x}-\hat{\mu}}$$

are used, which contain $O(a^2)$ errors. We will consider the effect of using

$$\tilde{\Delta}_\mu^{(+-)} = \Delta_\mu^{(+-)} - \frac{1}{6}\Delta^+ \Delta^{(+-)} \Delta^- \quad (10)$$

$$\tilde{\Delta}_\mu^{(2)} = \Delta_\mu^{(2)} - \frac{1}{12}\left[\Delta^+ \Delta^-\right]^2$$

which are correct up to $O(a^4)$. One can continue to correct to $O(a^6)$, we denote these derivatives $\tilde{\Delta}'^{(+-)}_\mu$ and $\tilde{\Delta}'^{(2)}_\mu$. As an additional constraint on c_A, we investigate whether Lorentz invariance is restored to sufficient accuracy i.e. that consistent values of m are obtained at zero and finite momentum.

3. Estimating C_A using excited states

The configurations and light quark propagators were provide by the UKQCD collaboration. The simulation parameters are given in table 1. For more details see reference [3]. Note that the C_{SW} values correspond to those determined by the ALPHA collaboration.

β	Volume	n_{confs}	C_{SW}	κ_l
6.0	$16^3 \times 48$	496	1.77	0.13344
6.2	$24^3 \times 48$	214	1.61	0.13460
β	κ_c	$\kappa_s(K)$	$a^{-1}(r_0)$	
6.0	$0.135252(^{+16}_{-9})$	$0.13401(^{+2}_{-2})$	2.12	
6.2	$0.135815(^{+17}_{-14})$	$0.13495(^{+2}_{-2})$	2.91	

Table 1
Simulation details.

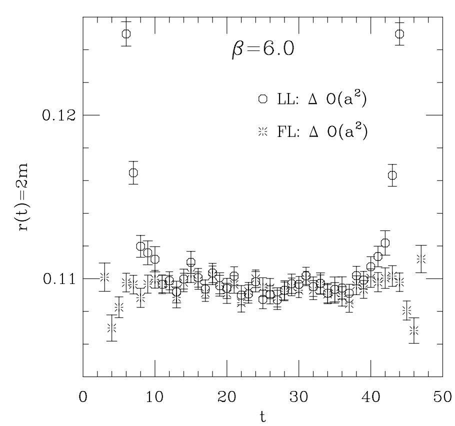

Figure 2. The ratio of correlators $r(t)$.

Figure 2 shows the ratio of correlators, $r(t)$, for the data set at $\beta = 6.0$ for correlators local at the source and sink (LL) and those fuzzed at the source and local at the sink (FL). The $O(a^2)$ lattice derivatives, equation 9, were used. Towards the center of the lattice $r(t)$ tends to a constant, $2m \sim 0.1096$. However, in the LL case, close to the origin there are significant deviations from this value due to discretisation effects. The much wider plateau in $r(t)$ for the FL correlators (where the fuzzing has been chosen to more or less eliminate excited state contributions) show that the discretisation effects in $r(t)$ LL are associated with excited states. We can determine c_A

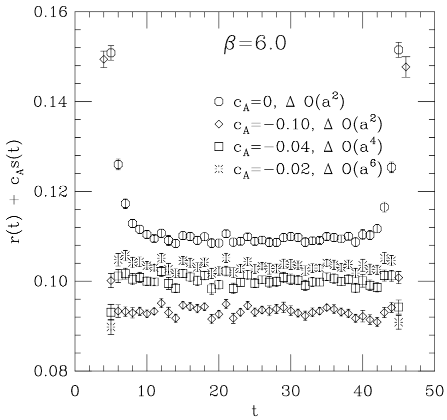

Figure 3. The $O(a)$-improved bare quark mass as determined using from LL correlators.

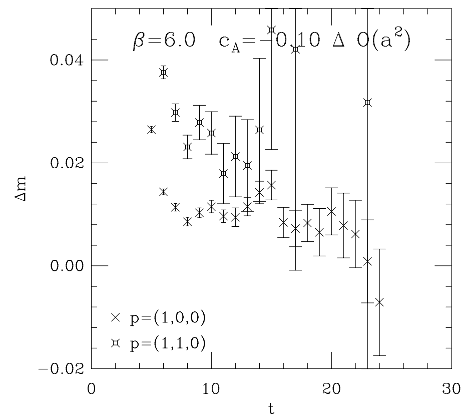

Figure 4. The difference in the bare quark mass determined at zero and finite momentum, where standard lattice derivatives are used.

by adding the term $c_A s(t)$ to reduce the discretisation effects at small t. To do this we perform a *correlated* fit to $r(t)$ using $r(t) = 2m - c_A s(t)$, to extract $2m$ and c_A. The values of c_A obtained are detailed in the next section.

To see how well the discretisation effects are reduced we plug these values back into $r(t) + c_A s(t)$ for LL correlators and compare with the unimproved case. At $\beta = 6.0$ we find that the plateau in $2m$ can be extended from $t = 11$ to $t = 6$ when $c_A = -0.10$ and $O(a^2)$ lattice derivatives are used. However, as shown in figure 3, if one improves ∂_μ, the discretisation effects can be removed by the same amount but lower values of c_A are required. Clearly the value of c_A and the form of the derivatives significantly affects the value of m extracted. Furthermore, we find that consistent values of m at zero and finite momentum can only be obtained if $O(a^4)$ or $O(a^6)$ derivatives are used. Figures 4 and 5 show the difference in the bare quark mass at zero and finite momentum for standard and improved derivatives.

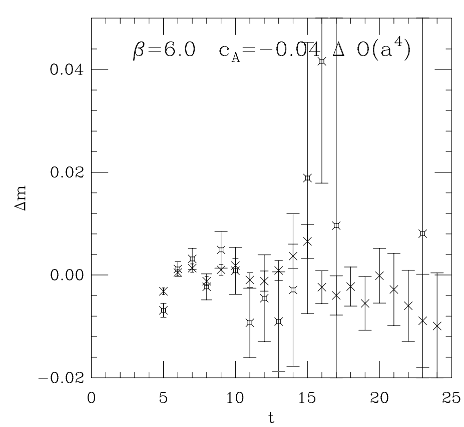

Figure 5. The difference in the bare quark mass determined at zero and finite momentum, where improved lattice derivatives are used.

4. Results for C_A

Results are shown in figures 6 and 7. The main points are:

- A reliable estimate for c_A is indicated by a stable value with t_{min}. At $\beta = 6.0$, the statistical errors grow rapidly for $t_{min} > 8$, although the results are reasonably stable for $t_{min} = 6-8$. The situation is similar at $\beta = 6.2$.

- Clearly c_A decreases when the temporal derivatives are improved. The decrease is less going from derivatives correct to $O(a^4)$ to those correct to $O(a^6)$, as expected.

- c_A obtained using improved derivatives is below the value obtained by the ALPHA collaboration. For the $O(a^6)$-correct derivatives c_A is close to the 1-loop tadpole-improved value (although the perturbation theory has not been performed with improved derivatives). In principle, our results can differ with those of reference [1] by $O(\Lambda_{QCD}a) \sim .300/2.1 = .14$ at $\beta = 6.0$ and .10 at $\beta = 6.2$.

- The results can be compared to those obtained by Bhattacharya et. al. [2]. They obtain $c_A = -0.02(2)(2)$ at $\beta = 6.0$ and $-0.033(4)(3)$ at $\beta = 6.2$ for κ_c with a significantly smaller number of configurations. Our results are at finite κ around $\kappa_{strange}$. A preliminary study indicates that the light quark mass dependence of c_A is small.

5. Conclusions

We have applied the method of reference [2] to extract the $O(a)$ improvement coefficient, c_A. We find that the accuracy and reliability of this method is limited by the small range of timeslices over which a stable value of c_A is found. Nevertheless, we have shown that the value of c_A obtained depends significantly on the discretisation chosen for the lattice derivatives appearing in the PCAC relation. c_A reduces as the lattice derivatives are improved, and lies between the value obtained by the ALPHA collaboration and the 1-loop tadpole-improved value. Furthermore, consistent values for the bare quark mass at zero and finite momentum can only be obtained for derivatives correct to $O(a^4)$ and above.

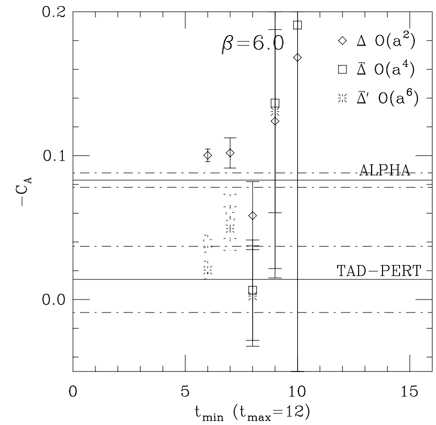

Figure 6. The results for c_A at $\beta = 6.0$ obtain using correlated fits with $Q > .10$ (the dashed points have $0.10 > Q > .01$). The data has been averaged over positive and negative (i.e. $t > 25$) timeslices. The errors are bootstrapped over 100 bootstrap samples. The result of the ALPHA collaboration is shown as a horizontal line, where the dashed lines indicate the numerical error. The tadpole-improved 1-loop perturbative result is also indicated, the error is taken to be $1\alpha_p^2(\pi/a)$.

These results suggest a similar study should be performed within the Schrödinger functional approach. In the future, we plan to investigate the quark mass dependence of c_A and check the scaling properties of the renormalised light quark mass and f_π obtained using our values of c_A.

6. Acknowledgements

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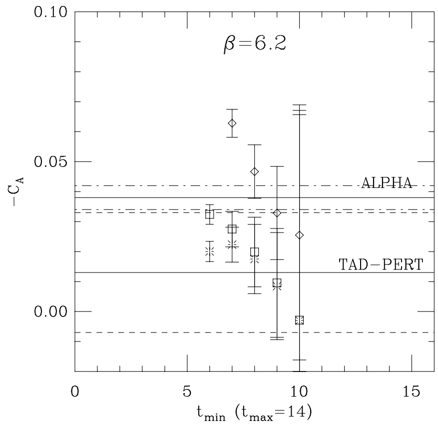


Figure 7. The results for c_A at $\beta = 6.2$. The figure is labelled in the same way as figure 6

3. UKQCD collaboration, K. Bowler et. al, Phys. Rev. D62 054506 (2000).

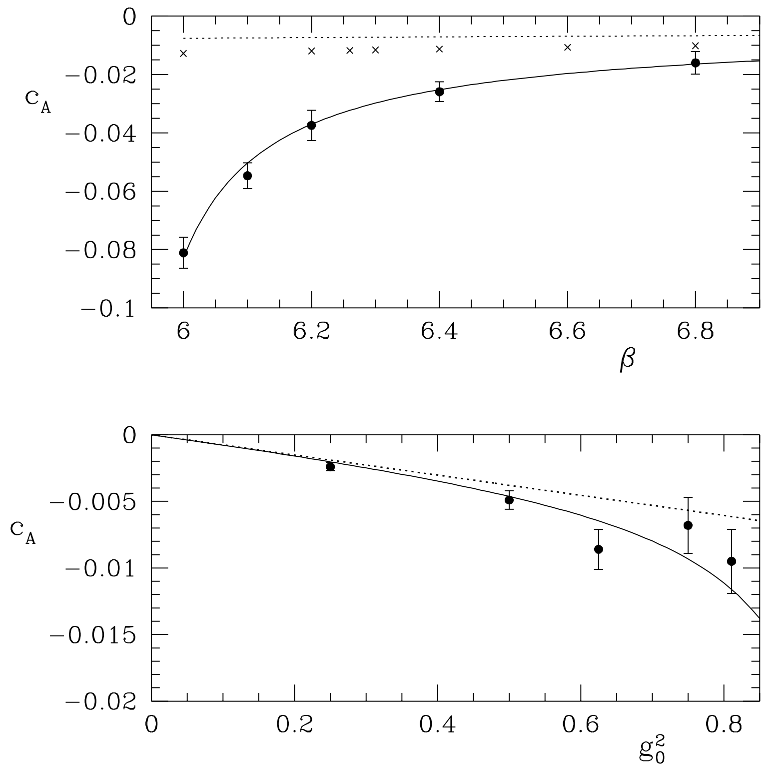


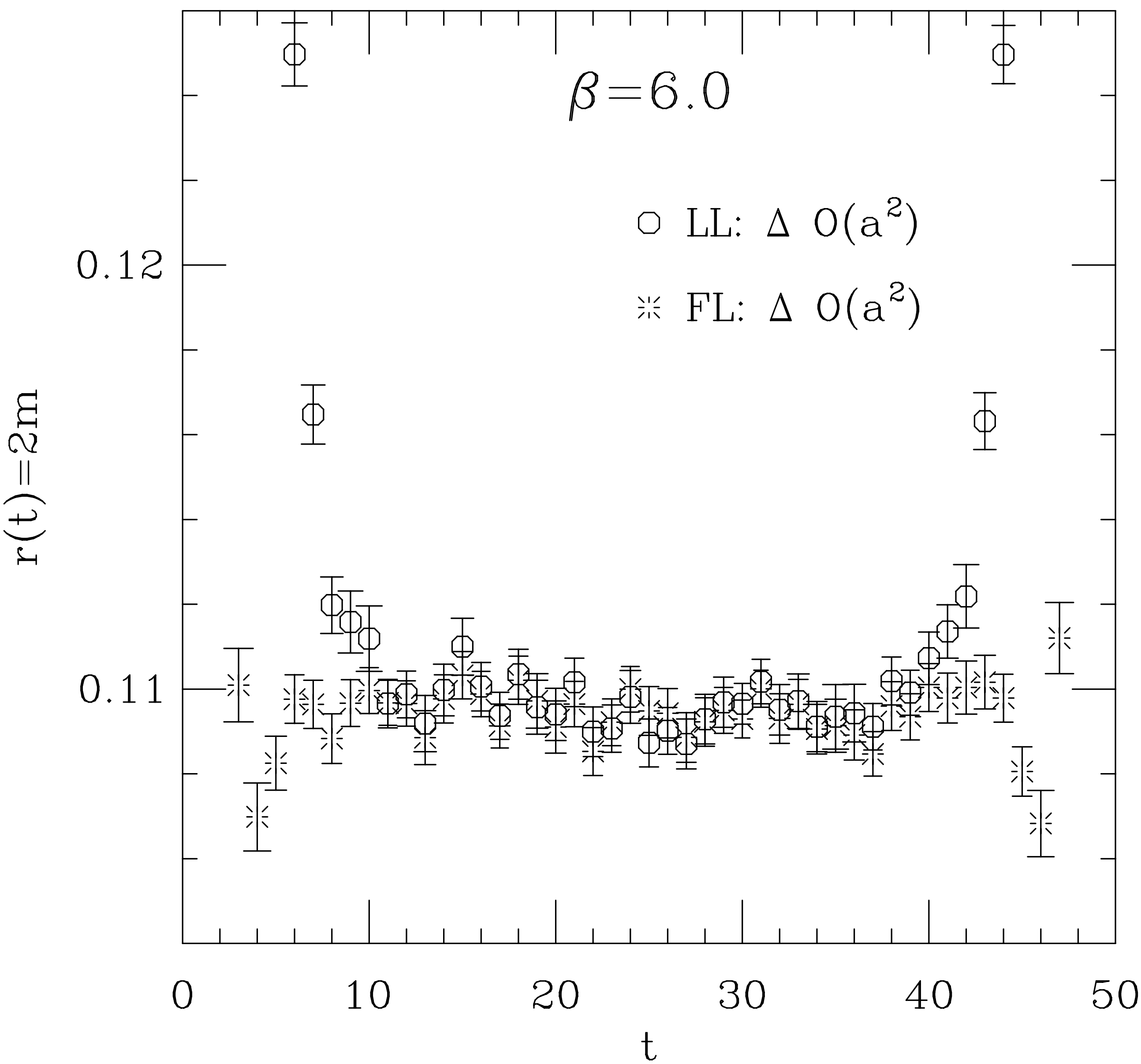
Fig. 5. Results for $c_{\rm A}$ from numerical simulations (filled circles), bare perturbation theory (dotted line) and "mean field improved" perturbation theory (crosses). The full line represents the fit (6.5).

from perturbation theory [4], $c_{\rm A}$ is rather small and remains so even at the largest values of the bare coupling considered. The one-loop formula,

$$c_{\rm A} = c_{\rm A}^{(1)} g_0^2 + {\rm O}(g_0^4), \qquad c_{\rm A}^{(1)} = -0.00756(1), \tag{6.4}$$

describes the data rather well for $g_0^2 \leq 0.5$, thus giving further evidence for the smallness of the residual cutoff effects in our determination of $c_{\rm A}$. "Mean field improved" perturbation theory here amounts to replacing g_0^2 by $g_{\rm P}^2$ [cf. eq. (5.14)], but as can be seen from fig. 5 this modification cannot make up for the large difference between perturbation theory and the data at low values of β.

For future applications it is again convenient to represent our results in



$\beta=6.0$
LL: Δ $O(a^2)$
FL: Δ $O(a^2)$
0.12
0.11
r(t)=2m
0
10
20
30
40
50
t